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ELASTOPLASTIC TORSION THEORY FOR WHISKER CRYSTALS

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1. Introduction. Plastic deformation of crystals is accompanied by the complex evolution of internal elastic stress fields caused by self-consistent movement of conglomerates of crystal defects at different structural levels of deformation [1]. A specific role in formation of these elastic fields applies to the crystal surface [2] which thus appears to be incorporated in a number of factors which affect movement of the defect structure. In view of this with the microscopic nature of some of the linear dimensions of a crystal in the kinetics of plastic strain the structural level of deformation, whose size is comparable with the size of a crystal, becomes decisive. In this respect whisker crystals (WC) are unique model objects for studying in a "pure form" features of the development of plastic deformation in assemblies of defects of different hierarchical degrees of structural levels. In an experimental respect torsion and bending are convenient methods for studying the ductility of WC [3], and the dislocation structure which forms is satisfactorily revealed by direct methods [4]. It is of interest to obtain general relationships which connect the macroscopic reaction of a WC (i.e., the amount of torsion) to dislocations present within it with characteristics of the dislocation structure which emerges as a basis for theoretical analysis of the plastic behavior of WC in torsion and also in the general case in bending.

In the present work in an approximation of macroscopically average description of the dislocation structure elastic torsion, and as a generalization bending, of whisker crystals caused by presence of dislocations in a crystal are considered. Relationships are found from the condition for minimum elastic energy which determine the macroscopic reaction of a whisker crystal to dislocations introduced into it.

2. Statement of the Problem. Finding actual elastic dislocation fields located close to a surface is a very complex mathematical problem [2] for which there is not yet a satisfactory solution. In particular, the problem of torsion for a WC containing dislocations is currently solved in a general form only for the case of rectilinear screw dislocations parallel to the crystal axis. As shown by Eshelby [2], the twist angle for WC ignoring edge effects at is ends is expressed in terms of the value of the Prandtl torsion function at

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points of the crystal cross section through which dislocations pass. The possibility of obtaining an analytical result in this case is explained by the two-dimensional nature of the elastic problem which arises due to translational invariance of the dislocation structure in question with respect to arbitrary displacement along the WC axis. It is possible to make progress on this question if macroscopically averaged description of the dislocation structure is used [5]. In view of this we turn attention to the fact that translational invariance is inherent for elastic stress fields in WC subject to torsion. With microscopic consideration of the dislocation structure forming in it with torsion it is naturally uniform along the crystal axis. However, as a result of external action, which is characterized by the property of translational invariance, on average it should be similar. Here longitudinal dimension $\Delta \ell$ of a physically infinitely small element of volume ΔV [5], for which averaging of the microscopic inhomogeneities of the dislocation structure which develops along the WC axis should be performed, may be related in different ways with its (crystal) transverse dimensions R. It is evident that the two-dimensional nature of the problem in question arises when

$$\Delta l \ll R. \tag{2.1}$$

In the opposite case the problem has a markedly three-dimensional character and analysis of it presents well-known difficulties.

Furthermore it is noted that torsional strain in a macroscopic sense is a special case of the more general deformation of rods with which there is rotation of neighboring rod cross sections around an arbitrary axis (in the case of torsion coinciding with the crystal axis). Vector $\mathbf{\Omega}$ serves as a macroscopic characteristic of this deformation whose component directed along the rod axis is the twist angle, and the component perpendicular to the axis characterizes rod bending so that in the general case [6]

$$\mathbf{\Omega} = d\boldsymbol{\varphi}/dl, \tag{2.2}$$

where $d\varphi$ is angle of rotation of two neighboring transverse rod cross sections separated from each other by distance $d\ell$ along the rod length. In the case of pure bending the elastic stressed state which arises in the rod does not depend on coordinates read along the rod axis [6], which together with (2.2) gives rise to physical analogy for these strains, and which suggests combined consideration of them.

Assuming that condition (2.1) is fulfilled, we consider the problem of torsion and pure bending for a WC with dislocations securely fastened at one end and subject at the other to the action of force moment M. As is well known from mechanics [7], reaction of a body to an external force does not depend on whether there are internal stresses in it or not. This situation makes it possible to distinguish torsion and bending $\mathbf{\Omega}^{(d)}$ caused by dislocation, and torison $\Omega_z^{(M)} = M_z/C$ and bending $\Omega_\alpha^{(M)} = I_{\alpha\beta}^{-1} M_\beta/E$ caused by an external moment (C is WC torsional stiffness, $EI_{\alpha\beta}$ is bending stiffness, E is Young's modulus, $I_{\alpha\beta}$ is tensor of moments of inertia for WC cross section [6] (here and subsequently Greek letters signify two-dimensional indices taking the values 1 and 2, and axis z of the coordinate system coincides with the crystal axis)). Vector $\Omega^{(d)}$ may be calculated by proceeding from the condition of minimum free energy for elastic deformation of a crystal in equilibrium with a fixed dislocation position. For this purpose it is only necessary to express this energy in the form of an appropriate functional for the WC cross section angle of rotation. It is noted that this statement of the problem, based on introducing vector Ω, in essence emerges from the semi-reverse St. Venant method [8], and as will be seen subsequently the assumption made above is justified.

<u>3. Elastic Energy of a WC with Dislocations as a Functional of the Macroscopic Parameters of Its Bending and Torsion.</u> In solving the variational problem it should be borne in mind that WC reaction to dislocations present within it comes down not only to torsion and bending (which is only the regular macroscopic part of this reaction), but also to macroscopic lattice distortion in the crystal surrounding the dislocation. Elastic energy E_0 connected with distortion does not depend on $\mathbf{\Omega}^{(d)}$ and therefore it may be omitted in the variational problem in question without detriment. In this case only the remaining part of E_{12} of the WC elastic strain energy is subject to analysis, and this is easy to establish by proceeding from the following reasoning in the spirit of Eshelby [7, 9]. Mentally we separate in a crystal of infinitely large dimensions the WC in question with dislocations and we cut it out by substituting the action of the rest of the crystal by assembled forces at the WC surface (it is necessary to close over the WC surface the ends of which emerge at the surface). With removal of these forces, which is equivalent to applying external surface forces of an opposite sign, as a result of internal stress relaxation the WC acquires some torsion

and it bends. Now by considering this torsion and bending as a result of external action we write energy E_{12} connected with it directly in the form of the sum of energy E_2 of elastic torsional and bending strains equal, respectively, to $C[\Omega_z^{(d)}]^2/2$ and $EI_{\alpha\beta}\Omega_\alpha^{(d)}\Omega_\beta^{(d)}/2$ calculated per unit length of WC [6], and the reaction energy E_1 corresponding to $\Omega^{(d)}$ of stress field p_{ik} with dislocations in the WC.

At first sight it may appear strange that in the expressions for E_{12} the term E_1 is considered which reflects the reaction between itnernal and external stresses since as is well known [7] the contribution of elastic energy in this reaction equals zero. However, it is necessary to bear in mind that stresses p_{ik} are not generated by sources external with respect to the WC, and they are only a regular macroscopic part of internal stresses of the dislocations themselves liberated on a microscopic background τ_{ik}. Therefore, in fact energy E_1 describes the reaction between two subsystems p_{ik} and τ_{ik} of internal stresses in a crystal caused by the same source, and consequently equality to zero of this energy does emerge from anywhere in the general case. In the mental experiment described above, which is nothing more than a convenient procedure, the occurrence of the second term in E_{12} is explained by the necessity in considering potential energy of including the "external" effect [7] which here is responsible for E_1 [7]. This point of view is convenient in the respect that in calculating E_1 it makes it possible to use a known equation directly for the energy of dislocation loop reaction with an external elastic stress field [7]. In view of the additive nature of the contribution made to the total reaction energy E_1 by all of the dislocations existing in the WC we write this energy as a sum

$$E_1 = -\left\langle \sum_d b_i^{(d)} \int_{S_d} p_{ik}\, dS_k \right\rangle. \tag{3.1}$$

In each of the terms presented integration is performed with respect to arbitrarily selected surface S_d resting on the corresponding dislocation loop d with Burgers vector $\mathbf{b}^{(d)}$,* and angular brackets signifying averaging values within them with respect to generalized coordinates of dislocations. By following the general scheme in [10] we consider a configuration space formed by population Q of generalized coordinates of all of the dislocations, and we introduce into it probability density f(Q) of configurational points which satisfy the normalization condition $\int f(Q)dQ = 1$. Then this operation of statistical averaging is accomplished formally by calculating the following integral with respect to the configurational dislocation space:

$$\langle \ldots \rangle = \int \ldots f(Q)dQ. \tag{3.2}$$

By means of the Dirac delta function at surface S_d [9]

$$\delta_i(S_d) = \int_{S_d} \delta(\mathbf{r} - \mathbf{r}')\, dS_i'$$

surface integrals which feature in Eq. (3.1) may be reduced to integrals taken with respect to volume V_0 of the WC. Then by changing in this equation the order of summing and integration we rewrite it in the form

$$E_1 = -\left\langle \int_{V_0} p_{ik} u_{ik}^{(s)}\, dV \right\rangle, \tag{3.3}$$

where the notation introduced in [6] is used

$$u_{ik}^{(s)} = \sum_d \frac{1}{2}\left[\delta_i(S_d)\, b_k^{(d)} + \delta_k(S_d)\, b_i^{(d)}\right]$$

where the δ-shape is a feature of the strain tensor formally relating to the stress field around a dislocatin separate at surface S_d. Since probability density f(Q) does not depend on spatial coordinates, then there is permutability of statistical averaging operation (3.2) with spatial differentiation and integration [10]. Considering this in (3.3) we write

$$E_1 = -\int_{V_0} p_{ik} \langle u_{ik}^{(s)} \rangle\, dV. \tag{3.4}$$

*In order to avoid misunderstandings connected with writing Eq. (3.1) it is noted that the definition of the Burgers dislocation vector used in this work [6] differs in sign from the definition adopted in [7].

First we consider slight bending of a WC (when the radius of curvature of the WC curved axis is considerably greater than its length [6]) by stipulating a particular case of severe bending. In fulfilling the smallness condition with respect to rotation of WC cross sections distant from each other by a distance of the order of its transverse dimensions, the dependence of elastic stresses p_{ik} on vector Ω is linear in nature:

$$p_{ik} = \chi_{ikm}\Omega_m. \tag{3.5}$$

According to the conclusions of classical torsion and slight bending theory for thin rods [6, 8] components which are uniquely distinguished from zero with respect to indices i and k of tensor χ_{ikm} are

$$\chi_{\alpha zz} = \chi_{z\alpha z} = 2\mu e_{z\alpha\beta}\partial\chi/\partial x_\beta, \quad \chi_{zz\alpha} = Ee_{z\alpha\beta}x_\beta. \tag{3.6}$$

Here μ is crystal shear modulus; $e_{ik\ell}$ is Levi-Civita tensor; χ is Prandtl stress function satisfying the equation $\partial^2\chi/\partial x^2 + \partial^2\chi/\partial y^2 = -1$ and boundary condition $\chi = 0$ at the contour of the WC cross section, and the origin of the coordinate system is selected at the center of inertia of the WC cross section:

$$\int\limits_{S_0} x_\alpha\, dx\, dy = 0. \tag{3.7}$$

Now by substituting (3.5) in (3.4) we separate in it integration with respect to area S_0 of WC cross section and we introduce the notation

$$M_k^{(d)} = \int\limits_{S_0} \chi_{ijk} \langle u_{ij}^{(s)} \rangle\, dx\, dy. \tag{3.8}$$

The reaction energy in question is presented in the form of an integral

$$E_1 = -\int\limits_0^{L_0} M_k^{(d)}\Omega_k^{(d)}\, dz,$$

taken for length L_0 of the WC. By adding to this energy with E_2 we have the following expression for the part of WC elastic strain energy sought connected with its macroscopic reaction on the dislocations present:

$$E_{12} = \int\limits_0^{L_0} \left\{\frac{1}{2} C\, [\Omega_z^{(d)}]^2 + \frac{1}{2} EI_{\alpha\beta}\Omega_\alpha^{(d)}\Omega_\beta^{(d)} - M_k^{(d)}\Omega_k^{(d)}\right\} dz. \tag{3.9}$$

4. Solution of the Variational Problem of Macroscopic Bending and Torsion of a WC with Dislocations. From the condition of minimum energy (3.9) in equilibrium by varying with respect to vector components the rotation angle $\varphi^{(d)}$ for WC transferse sections taking account of (2.2) we have [6]

$$(EI_{\alpha\beta}\Omega_\beta^{(d)} - M_\alpha^{(d)})\,\delta\varphi_\alpha^{(d)} - \int\limits_0^{L_0} \frac{d}{dz}(EI_{\alpha\beta}\Omega_\beta^{(d)} - M_\alpha^{(d)})\,\delta\varphi_\alpha^{(d)}\, dz +$$
$$+ (C\Omega_z^{(d)} - M_z^{(d)})\,\delta\varphi_z^{(d)} - \int\limits_0^{L_0} \frac{d}{dz}(C\Omega_z^{(d)} - M_z^{(d)})\,\delta\varphi_z^{(d)}\, dz = 0,$$

where in the first and third terms their value is taken at the unsecured end of the WC. In view of arbitrary variation of $\delta\varphi_k^{(d)}$ both along the WC length and at the unsecured end from the equality it follows that minimum energy E_{12} is reached with

$$\Omega_z^{(d)} = M_z^{(d)}/C,\ \Omega_\alpha^{(d)} = I_{\alpha\beta}^{-1}M_\beta^{(d)}/E. \tag{4.1}$$

Furthermore we introduce (generally speaking asymmetrically) tensor J_{ik} according to the definition

$$J_{zz} = \frac{4\mu}{E}\chi,\ e_{z\delta\beta}\frac{\partial J_{\beta\alpha}}{\partial x_\delta} = e_{z\alpha\beta}x_\beta,\ J_{\alpha z} = J_{z\alpha} = 0, \tag{4.2}$$

by means of which we write (3.6) as

$$\chi_{ijk} = E\delta_{z(i}e_{j)\alpha n}\partial J_{nk}/\partial x_\alpha. \tag{4.3}$$

Now by substituting (4.3) in (3.8) we find an equation for $M^{(d)}$ in the form

$$M_i^{(d)} = E\int\limits_{S_0} e_{k\alpha n}\frac{\partial}{\partial x_\alpha}\left(J_{ni}\langle u_{kz}^{(s)}\rangle\right)dx\,dy - E\int\limits_{S_0} J_{ni}e_{k\alpha n}\frac{\partial\langle u_{kz}^{(s)}\rangle}{\partial x_\alpha}dx\,dy.$$

Here the first integral, being transformed by the Stokes theorem [6] into an integral for the length of the contour of the WC cross section, disappears in view of boundary conditions $\chi = 0$ at this contour and with fulfillment of the relationship

$$e_{z\alpha\beta}J_{\alpha\delta}n_\beta = 0, \tag{4.4}$$

where $\mathbf{n}$ is vector of the external normal to the contour line of the cross section. Consideration of the second integral leads to the equation

$$M_i^{(d)} = E\int\limits_{S_0} J_{ni}e_{n\alpha k}\frac{\partial\langle u_{kz}^{(s)}\rangle}{\partial x_\alpha}dx\,dy. \tag{4.5}$$

We transform it by using determination of the dislocation density tensor:

$$\rho_{ik} = e_{ilm}\partial u'^{(s)}_{mk}/\partial x_l, \tag{4.6}$$

which corresponds to determination of this value adopted in [6]. Here

$$u'^{(s)}_{mk} = u^{(s)}_{mk} + e_{mkj}\omega_j^{(s)} \tag{4.7}$$

is the basic field of plastic distortion including plastic rotation

$$\omega_j^{(s)} = \sum_d \frac{1}{2}e_{jmk}\delta_m(S_d)\,b_k^{(d)}.$$

By substituting (4.7) in (4.6) we find that

$$e_{ilm}\partial u^{(s)}_{mk}/\partial x_l = \partial\omega_i^{(s)}/\partial x_k + K_{ki}, \tag{4.8}$$

where the second term in the right-hand part is the tensor of Nye curvature [9]:

$$K_{ki} = \rho_{ik} - (1/2)\delta_{ik}\rho_{ll}. \tag{4.9}$$

By assuming in (4.8) that k = 3 and averaging taking account of the permutability noted above for this operation with spatial differentiation we obtain

$$e_{ilm}\partial\langle u_{mz}^{(s)}\rangle/\partial x_l = \partial\langle\omega_i^{(s)}\rangle/\partial z + \langle K_{zi}\rangle. \tag{4.10}$$

In view of the suggested translational invariance of the average dislocation structure values $\langle\omega_i^{(s)}\rangle$ and $\langle u_{ik}^{(s)}\rangle$ do not depend on coordinate z, and consequently the corresponding partial derivatives in (4.10) equal zero. Finally Eq. (4.5) taking account of (4.2) may be written in the form

$$\begin{aligned}\Omega_z^{(d)} &= \frac{4\mu}{C}\int\limits_{S_0}\chi\langle K_{zz}\rangle\,dx\,dy,\\ \Omega_\alpha^{(d)} &= I_{\alpha\beta}^{-1}\int\limits_{S_0}J_{\delta\beta}\langle K_{z\delta}\rangle\,dx\,dy.\end{aligned} \tag{4.11}$$

The tensor $J_{\alpha\beta}$ used in (4.11) is determined by Eqs. (4.2) and boundary conditions (4.4) unambiguously since the number of independent components of tensor $J_{\alpha\beta}$ exceeds the number of Eqs. (4.2). However, this is arbitrary in determining $J_{\alpha\beta}$ in view of the relationship

$$\partial\langle K_{z\alpha}\rangle/\partial x_\alpha = 0, \tag{4.12}$$

which emerges from (4.6) and (4.9) and is a consequence of the generalized conservation rule for the Burgers vector [5, 6], and it does not affect the result of using (4.11). This is easy to demonstrate by using the identity

$$J_{\delta\beta}\langle K_{z\delta}\rangle = e_{z\alpha\delta}\frac{\partial}{\partial x_\alpha}\left(J_{\delta\beta}\int\limits_{\mathbf{r}_0}^{\mathbf{r}} e_{z\varepsilon\gamma}\langle K_{z\gamma}\rangle\,dx'_\varepsilon\right) - e_{z\alpha\delta}\frac{\partial J_{\delta\beta}}{\partial x_\alpha}\int\limits_{\mathbf{r}_0}^{\mathbf{r}} e_{z\varepsilon\gamma}\langle K_{z\gamma}\rangle\,dx'_\varepsilon, \tag{4.13}$$

where the integrability of form $e_{z\varepsilon\gamma}\langle K_{z\gamma}\rangle dx'_\varepsilon$ provides fulfillment of condition (4.12); $\mathbf{r}_0$ is an arbitrary vector in the plane of cross section. By substituting (4.13) in (4.11) and transforming the first integral by means of the Stokes theorem into an integral with respect to the line of the contour of the cross section, which disappears in view of (4.4), taking account of (4.2) we have

$$\Omega_\alpha^{(d)} = I_{\alpha\delta}^{-1} \int\limits_{S_0} e_{z\delta\beta} x_\beta \left(\int\limits_{r_0}^{r} e_{z\gamma\varepsilon} \langle K_{z\gamma} \rangle \, dx'_\varepsilon \right) dx\, dy.$$

According to (4.12) the functional obtained does not depend on the integration path in the internal integral, and in view of (3.7) due to selection of r_0 the $\Omega_\alpha^{(d)}$ is determined with respect to $\langle K_{z\beta} \rangle$ unambiguously in this expression, and consequently in Eq. (4.11). Thus, relationship (4.11) establishes the connection sought between averaged dislocation structure characteristic (Nye curvature tensor) and the macroscopic reaction of the crystal to dislocations introduced into it, and this reaction does not depend on the plastic prehistory of the crystal, but it is only determined by the specific dislocation distribution.

5. Illustrative Examples. As important examples we consider some specific solutions.

Dislocations parallel to the WC axis are characterized by generalized coordinate ρ_0 which is the radius-vector of a point at the plane of WC cross section through which a dislocation passes. Since the generalized coordinate in this case is invariant with respect to transfer of the coordinate system along the WC axis, then as a function of f(Q) it is possible to adopt $\delta(\rho_0 - \rho_0')$. Then by taking account of (3.2) and (4.9), and considering that the vector of the tangent to the line of the dislocation is antiparallel to its Burgers vector, for a screw dislocation we obtain $K_{zz} = -(b/2)\delta(\rho - \rho_0)$. Then from (4.11) it follows that $\Omega_z^{(d)} = -(2\mu b/C)\chi(\rho_0)$, i.e., the well-known Eshelby result [2] for an axial screw dislocation. An axial edge dislocation only has components $K_{\alpha z}$ differing from zero, and according to (4.11) it does not create bending and torsion which was also noted in [2].

In the case of a constant dislocation density taking account of the normalizing condition for function f(Q) it emerges that $\langle K_{zz} \rangle = (1/2)(\rho_{zz} - \rho_{xx} - \rho_{yy}) = \mathrm{const}$, $\langle K_{z\alpha} \rangle = \rho_{\alpha z} = \mathrm{const}$. Since $C = 4\mu \int \chi dx\, dy$, then the first equation of (4.11) takes the form $\Omega_z^{(d)} = (1/2)(\rho_{zz} - \rho_{xx} - \rho_{yy})$. Whence torsion for screw dislocations parallel to the WC axis and having density ρ equals $\Omega_z^{(d)} = -\rho b/2$, for dislocations perpendicular to the WC axis $\Omega_z^{(d)} = \rho b/2$, and it equals zero with the condition $\rho_{zz} = \rho_{xx} + \rho_{yy}$ (for screw dislocations at an angle of 45° to the crystal axis). In order to transform the second of relationships (4.11) it is necessary to multiply (4.2) by x_ε and to integrate the expression to the left in parts:

$$\int\limits_{S_0} e_{z\delta\beta} \frac{\partial (J_{\beta\alpha} x_\varepsilon)}{\partial x_\delta} dx\, dy - \int\limits_{S_0} e_{z\varepsilon\beta} J_{\beta\alpha} \, dx\, dy = \int\limits_{S_0} e_{z\alpha\beta} x_\beta x_\varepsilon \, dx\, dy.$$

The first integral is transformed into an integral with respect to the contour line of the cross section and it disappears in view of (4.4) so that

$$\int\limits_{S_0} J_{\alpha\beta} \, dx\, dy = \int\limits_{S_0} e_{z\alpha\delta} e_{z\beta\gamma} x_\delta x_\gamma \, dx\, dy = I_{\alpha\beta}.$$

Thus, for a macroscopic uniform dislocation density there is the relationship $\Omega_k^{(d)} = \langle K_{zk} \rangle$ which conforms with the known result of Nye [7, 9].

In conclusion it is noted that a change-over to the case of severe bending may be accomplished in a similar way to that in [6] by introducing (local) coordinate systems in each WC cross section parallel to each other and an original system in an undeformed crystal rotating together with cross sections with WC bending and twisting. Relationships (4.1) and (4.5), and consequently also (4.11) considered in a local coordinate system in the vicinity of a given WC cross section, remain valid with severe crystal bending. In contrast to bending under the action of an external moment [6], dislocation bending of a WC is not accompanied by additional WC twisting. This situation is due to the fact that vector $M^{(d)}$ turns together with rotation of the cross section, whereas moment M retains its orientation unchanged in space.

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NUMERICAL STUDIES OF NONLINEAR WAVE PROCESSES IN A LIQUID AND A DEFORMABLE SOLID DURING HIGH-SPEED IMPACT INTERACTION

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Problems of hydrodynamic shock loading of deformable bodies are most often encountered in the study and prevention of the erosional failure of structures interacting with a liquid. Among the structures that are subject to high-speed shock loading by liquid particles are turbine blades operating in moist vapor and elements of air and space craft flying in rain or entering bodies of water. Bodies immersed in a cavitating liquid are also subjected to shock-wave loading. The local pressures on the surface of solids involved in such interactions may exceed thousands of atmospheres [1]. There is yet another interesting aspect of such problems – the need to intensify the destructive effects achieved in the hydrodynamic extraction of minerals and fracture of rocks and the development of progressive new methods of cutting materials.

In order to protect structures from failure and select the proper materials and coatings, it is necessary to perform a detailed analysis of their deformation and fracture with different rates of interaction with liquids. The capabilities of empirical methods are extremely limited, since these interactions are of a drop- or jet-mediated nature (with the jets being of the shaped charge type) and are highly localized – with a duration measured in microseconds. The results that have been obtained through experimentation are for the most part qualitative. Only the ablation rate in such interactions provides quantitative data from such studies [1, 2]. The possibilities of theoretical investigations are even more limited. In mathematical modeling the high-speed impact interaction of bodies with a liquid, it is necessary to consider the compressibility of the media, the propagation of shock waves (SW) in them, the nonlinear behavior of the materials (dependent on the loading rate), and the resistance of the materials to plastic shears. The presence of the free surface of the liquid – which changes during the interaction – complicates the solution of the problem [2, 3].

Only a small number of studies [2, 4-6, etc.] have numerically investigated features of the nonlinear deformation and fracture of bodies in such interactions with a liquid. All of them are based on simplifying assumptions made relative to the behavior of the media. However, as was noted in [7], the use of such assumptions makes it possible to determine features of flow in the liquid that are important in determining the loading, deformation, and mode of failure of the given body. The most thorough studies of the dynamics of a drop liquid were made in [7], although they were limited to modeling flows in a liquid in the case of a collision with a nondeformable surface.

The present investigation, being a continuation of [8, 9], numerically examines wave processes in a drop liquid and a deformable body during their high-speed collision. The results are obtained with allowance for the above-mentioned features of the behavior of the media on the basis of the finite differences method and a through computing scheme of the Lachs–Vandroff predictor-corrector type. The Boris–Book flux correction method [10] is

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